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Correlations in financial time series: established versus emerging markets

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Abstract. Long-time correlations in both well-developed and emerging market indexes are studied. The Hurst exponent as well as detrended fluctuations analysis (DFA) are used as technical tools. Some features that seem to be specific for developing markets are discovered and briefly discussed.

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1 Introduction

The purpose of this work is to investigate if there are some long-time correlations in some typical financial time series, namely in the daily values of some well-known indexes. Apart from the most popular indexes such as DJIA (New York) and DAX (Frankfurt), we also consider indexes of three emerging markets: WIG (Warsaw), BUX (Budapest), and PS (Prague). We hope to find some crucial differences that could be uniquely attributed to the developing economies of Poland, Hungary, and Czech Republic, in contrast to the well-established markets of USA and Germany.

First we compute the Hurst exponent H *via* an interpolation method. Then we find this exponent once again using the empirical method based on the R/S parameter. This approach gives us a better control over accuracy for much shorter time-series that are available for emerging markets (WIG - 1727 observations, BUX - 958 observation, PS - 901 observation). For DJIA and DAX we use much longer sets of data (18 500 and 7253 observations, respectively). Next, for the same sets of data, we calculate the parameter $\alpha_{\rm dfa}$ using the detrended fluctuations analysis (DFA) method. It will serve as a verification of the Hurst method. If both R/S analysis and DFA will show some long-term correlations in the investigated time series, we can be more convinced that there is something real behind our results. If there is, however, a disagreement between these two methods, we will rather follow suggestions of Vandervalle and Ausloos [1] that the R/S analysis gives less trustworthy results than DFA (whatever the reason can be, *e.g.*, too short data set available).

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2 R/S analysis – Hurst exponent

Hurst observed that many natural phenomena follow a biased random walk. He proposed to measure the strength of trends in such processes by the value of a certain exponent. Generally speaking, the Hurst exponent is a measure of the rate of change of rescaled range with change of the length of time over which measurements are performed. Let N be the time series' length. We divide it into d sub-series of length n, so that $dn = N$. For each $m = 1, 2, \ldots, d$ and $t = 1, \ldots, n$ let

$$X_{t,m} = \sum_{u=1}^{t} (Z_{u,m} - M_m) \tag{1}$$

be a time series containing n observations, where $Z_{u,m}$ is the change within the period m, M_m is the average change within the same period m, $X_{t,m}$ is the cumulative deviation in this period. Thus the range is given as

$$R = \max(X_{t,m}) - \min(X_{t,m}). \tag{2}$$

To compare different time series we have to divide the above range by the standard deviation of the original data, obtaining $(R/S)_m$. This procedure is then iterated for all m. As a result the average R/S for a given t is calculated. The relation between the length of time period n and the rescaled range is

$$R/S = (a\ n)^H, \tag{3}$$

where H is the Hurst exponent.

Interpretation of this expression comes from the statistical mechanics. If the investigated time series is described by a random walk, H should be close to 0.5 and, consequently, the rescaled range grows proportionally to square root of time (N in our case). This means that the observations in the time series are independent and, possibly, have the Gauss distribution. If, on the contrary, the observed value of the Hurst exponent H is bigger than 0.5, then the observations are all dependent and the process itself can be described as the fractional Brown motion. Such a process exhibits a trend. Its strength grows with the value of H approaching 1. A time series with such properties is called persistent and can exhibit fractal properties.

Practical calculations of the Hurst exponent are somewhat difficult. The simplest method is, perhaps, the linear regression after taking the logarithms of both sides of (3). Using

$$S_t = \ln(\frac{P_t}{P_{t-1}}) \tag{4}$$

and applying (1) and (2) for different time increments (here for different lengths of n) we obtain rescaled ranges R/S, and then calculate the parameter H from the following equation

$$\log(R/S) = H \ \log(n) + \log(a). \tag{5}$$

It should be stressed, however, that in the case of too short data sets the above procedure can be problematic.

3 Detrended fluctuation analysis

Here we use the DFA method to verify the results from the previous section. Description of the DFA algorithm is given in [1–3]. We follow here the approach of Vandevalle and Ausloos [1]. Let $y(n)$ be a time series of length N. We subdivide the series into N/t non-overlapping sub-series of length t. In each of them we eliminate the linear trend $z(n) = an + b$, where parameters a and b are fitted by means of the least squares method. Next we calculate the DFA function:

$$F(t) = \sqrt{\frac{1}{t}\sum_{n=k\ t+1}^{((k+1)t}(y(n)-z(n))^2} \tag{6}$$

where $k = 1, 2, \ldots , (N/t - 1)$. Averaging $F(t)$ over N/t subsets, gives the mean value of $\langle F(t)\rangle$ for a given subseries length t. If we observe the linear dependence looking at the double logarithmic plot, the linear coefficient will estimate a value of the desired parameter, which we call α_{dfa}. Formally, we expect the following relationship to hold

$$\langle F(t)\rangle = a \exp(\alpha_{\mathrm{dfa}}\ t), \tag{7}$$

which, after taking logarithms on both sides, gives

$$\log(\langle F(t)\rangle) = \alpha_{\mathrm{dfa}} \log(t) + \text{const.} \tag{8}$$

Table 1. The values of the Hurst exponents and α_{dfa} parameters for stock-market indexes (DJIA, DAX, BUX, WIG, PS). Hurst exponents are calculated by an interpolation method. In the last column YES means that the Hurst analysis and DFA lead to similar conclusions. The answer NO means just the opposite.

Name	H - interpolation	DFA-αdfa	Does DFA and R/S confirm each other?
DJIA	0.54	0.51	yes
DAX	0.51	0.52	yes
BUX	0.58	0.56	yes
WIG	0.69	0.52	no
PS	0.6	0.56	no

Similarly to the Hurst analysis, interpretation of the results depends on the value of α_{dfa}. In the simplest case of the one dimensional Brownian motion, α_{dfa} is equal to 0.5 (just as in the case of the Hurst exponent). This method has advantages comparing to R/S analysis: (i) eliminating the linear trends one avoids some artifacts related to their existence and (ii) DFA analysis allows to avoid detecting any spurious long-time correlations (*e.g.*, due to non-stationary character of a time series).

4 Results and discussion

The main results obtained by our analysis are summarized in the Table 1 and in Figures 1 and 2. It is clearly seen that in all considered cases the Hurst exponents are different from 0.5. Thus the investigated indexes do not follow the simple random walk. A more or less striking persistence is observed. However for most of the series, the DFA does not fully support the Hurst analysis: the parameter α_{dfa} is rather close to 0.5. For BUX and PS this parameter is apparently greater than 0.5, but we can see from Figure 2 that just in these cases the relationship between $\log F(t)$ and $\log t$ deviates from the linear form. This can be caused by a relatively small number of available data. The error of estimation of α_{dfa} is, therefore, quite large. We see that the Hurst exponents are apparently greater for the emerging markets. We believe that a more sophisticated analysis along the lines presented in this paper will eventually reveal some special features helping to quantify differences between well-developed and emerging markets.

5 Brief summary

Long-time correlations in both well-developed and emerging market indexes (DJIA, DAX, WIG, BUX, PS) were investigated *via* the Hurst exponent as well as the detrended fluctuations analysis. In contrast to DJIA and DAX which

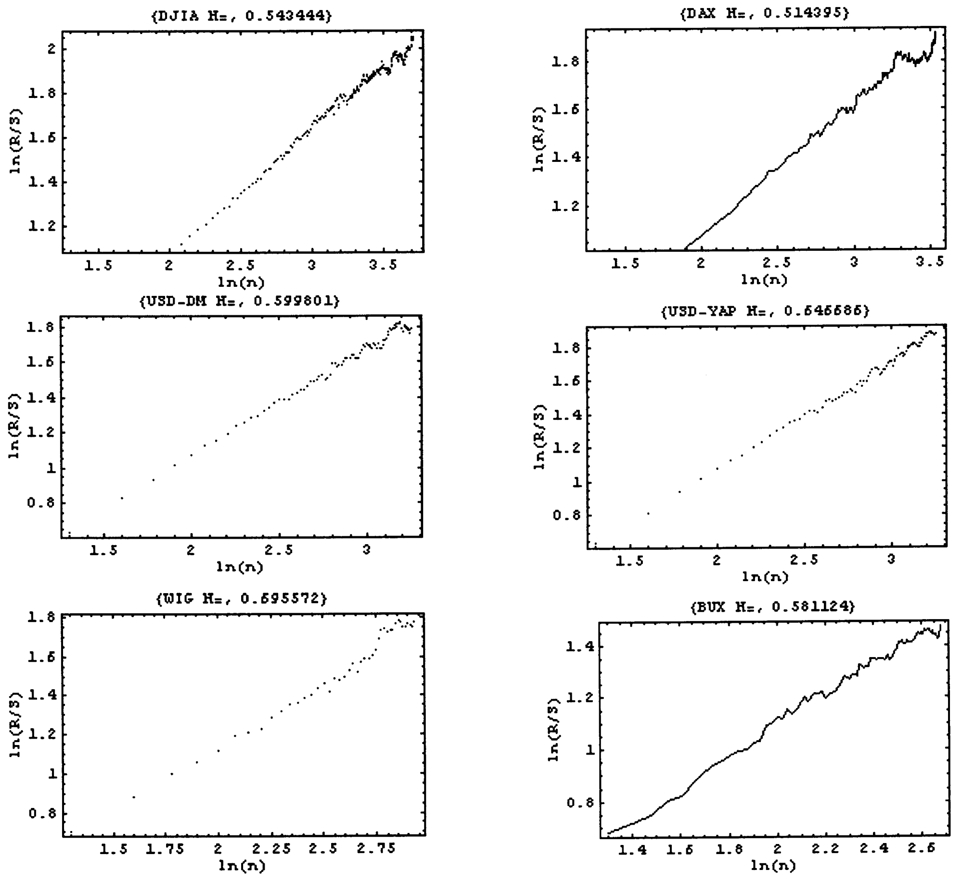

Fig. 1. Graphical presentation of the R/S analysis performed to infer the values of the Hurst exponents. Apart from stock-market indexes (DJIA, DAX, WIG, BUX) the analysis of the rate of exchange for USD-DEM and USD-JPY is given for comparison.

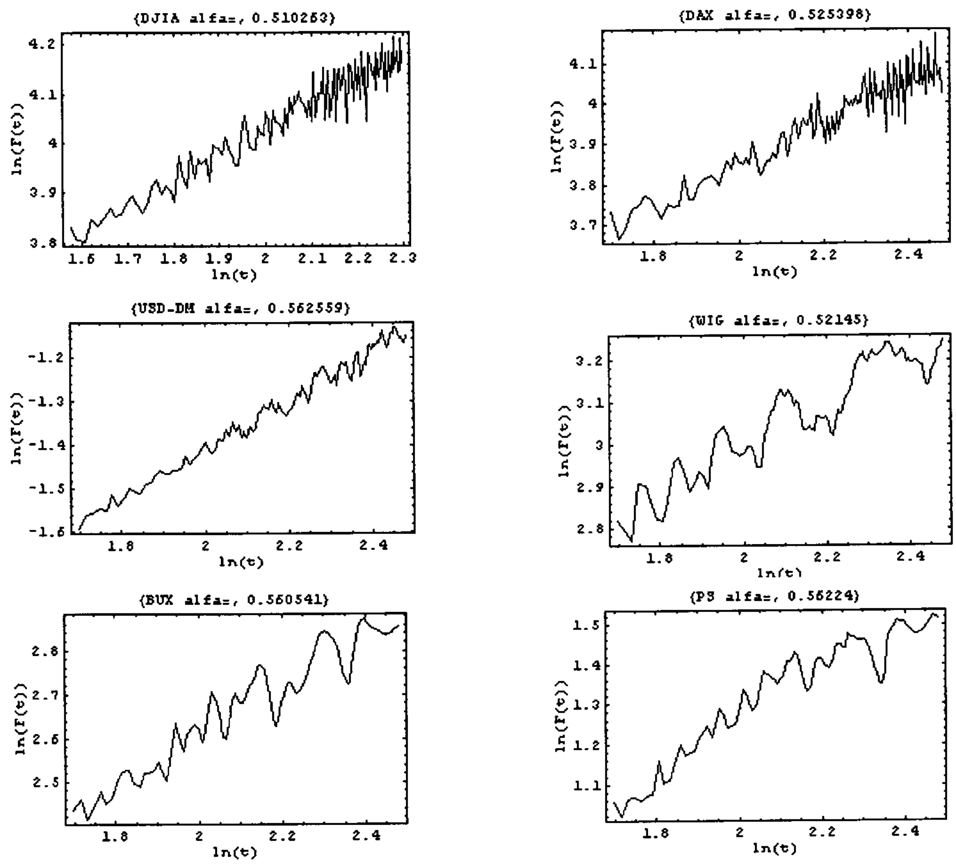

Fig. 2. Graphical presentation of detrended fluctuations analysis for stock-market indexes (DJIA, DAX, WIG, BUX, PS). A plot corresponding to the rate of exchange for USD-DEM is included for comparison.

do not show too strong persistent behavior, the Hurst exponents for emerging-market indexes are greater than 0.5, which may be a sign for their fractal structure. Despite the unavoidable inaccuracies caused by the relatively small size of data sets available for emerging markets, we believe that our observations do exhibits some features that are peculiar for East European developing markets. We plan to extend our analysis and to provide a better evidence for our conjectures.

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